9/16/25

Lecture 7

Proof by contradiction:

If the product of two positive numbers < 400, then at least one of the two nums > 20.

This is what I want to prove

Proof by contradiction

- For proof by contradiction, assume what you want to prove is false.
  - Assume both nums ≤ 20

$$\left.\begin{array}{lll} x \leq 20 & \Rightarrow & xy \leq 20y \\ y \leq 20 & \Rightarrow & 20y \leq 20 \cdot 20 \end{array}\right\} \Rightarrow xy \leq 400$$

Contradiction

☆ Logic behind the proof by contradiction:

start: $p \rightarrow q$

negate: $\neg(p \rightarrow q)$

conditional identity: $\neg(\neg p \vee q)$

deMorgan Law: $(p \wedge \neg q)$ ← show that this will lead to a contradiction.

☆ For proof by contradiction:

- start with the statement
- negate the statement
- show the negation is always false (contradiction)

Proof by contradiction example:

"$\sqrt{2}$ is irrational"

- Assume $\sqrt{2}$ is rational
- $\sqrt{2} = \frac{a}{b}$, $b \neq 0$
- $\sqrt{2} = \frac{m}{n}$, $n \neq 0$, $GCD(m,n) = 1$
  - $2 = \frac{m^2}{n^2}$
  - $2n^2 = m^2$
  - Axiom: m is even because m^2 is even
  - $m = 2k \quad k \in \mathbb{Z}$
  - $2 = \frac{(2k)^2}{n^2}$
  - $n^2 = \frac{(2k)^2}{2}$
  - $n^2 = 2k^2$ • Axiom: any integer multiplied by 2 is even
  - n is even

Proof by Cases

$\boxed{\text{Example!}}$

$x \in \mathbb{R}, y \in \mathbb{R} \qquad |x-y| \leq |x| + |y|$

Case 1 $\quad x+y \geq 0$: $|x+y| = x+y \leq |x| + |y|$

$\left.\begin{array}{l} x \leq |x| \\ y \leq |y| \end{array}\right\} \rightarrow x+y \leq |x|+|y|$

case 2 $\quad x+y < 0$: $|x+y| = -(x+y) \leq |x| + |y|$

$= -x - y \leq |x| + |y|$

$\left.\begin{array}{l} -x \leq |x| \\ -y \leq |y| \end{array}\right\} \rightarrow -x - y \leq |x|+|y|$